Uniform routings of shortest paths in graphs with large automorphism groups

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A *routing* R of a given connected graph Γ of order n is a collection of $n(n-1)$ paths connecting every ordered pair of distinct vertices of Γ. If these paths are the shortest, R is called a *routing of shortest paths.* The *load* $\xi(\Gamma, R, v)$ of a given vertex v is the number of paths of R passing (not beginning or ending) through v. When designing networks, it is often beneficial when every vertex has the same load. Such a routing is called a *uniform routing of shortest paths*, or shortly URSP.

Many vertex-transitive (v-t) graphs have been shown to admit a URSP. For example, all Cayley and quasi-Cayley graphs, i.e. all graphs containing a regular set of automorphisms [1]. However, not all v-t graphs belong in this class - the underlying graph of the dodecahedron does not admit a URSP [2]. The connection between the level of 'symmetry' of a graph and the existence of a URSP is not well understood. We have shown that vertex-transitivity is not necessary for the existence of a URSP. However, in the case of geodetic graphs, i.e. graphs having precisely one shortest path between any two vertices, we have shown that vertex-transitivity suffices. We will present a classification of planar geodetic graphs with respect to the existence of URSP.

References

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[2] S.Shim, J.Širáň, and J.Žerovnik, Counterexamples to the uniform shortest path routing conjecture for vertex-transitive graphs. *Discrete Appl. Math.* 119.3, 2002, pp.281–286.