An algorithmic proof for the domination number of grid graphs

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The *domination number* $\gamma(G)$ of a graph G is the minimum cardinality of a dominating set of G. Let the notion $[i]$ denote the set $\{1, 2, \ldots, i\}$. Let $G_{m,n}$ denote the complete (m, n) grid; i.e., the vertex set of $G_{m,n}$ is $[m] \times [n]$, and two vertices (i, j) and (i', j') are adjacent if $|i - i'| + |j - j'| = 1$. Reference [1] proves that for every $16 \leq m \leq n$, $\gamma(G_{m,n}) = \left\lfloor \frac{(m+2)(n+2)}{5} \right\rfloor - 4$ and thus concludes the calculation of $\gamma(G_{m,n})$ of all (m, n) grid graphs. In this study (a preliminary version was in [2]), we consider the charging pad deployment problem (CPDP) for wireless rechargeable sensor networks with grid topology $\mathcal{G}_{m,n}$. CPDP aims to find a deployment of charging pads for the unmanned aerial vehicle (UAV) so that every sensor node is covered by at least one pad and the number of pads is as small as possible. Note that $\mathcal{G}_{m,n} \cong G_{m+1,n+1}$. We show that when the grid length $\mathcal{L}$ satisfies $\frac{1}{\sqrt{2}} d_\theta < \mathcal{L} \leq \frac{2}{\sqrt{5}} d_\theta$, CPDP is related to finding $\gamma(\mathcal{G}_{m,n})$, where d_θ is the maximum flying distance of the UAV when its energy is θ (a pre-defined energy threshold). We propose a charging pad deployment scheme for $\mathcal{G}_{m,n}$ and prove that for every $15 \leq m \leq n$, our scheme uses the least number of pads; this provides an algorithmic proof for the domination number of grid graphs.

References

[1] D. Gonçalves, A. Pinlou, M. Rao, and S. Thomassé, The domination number of grids, *SIAM J. Discrete Math.*, vol. 25, no. 3, pp. 1443–1453, 2011.

[2] Y.-T. Shen, *Three charging pad deployment schemes for wireless rechargeable sensor networks with grid topology*, (Master thesis), National Yang Ming Chiao Tung University, Hsinchu, Taiwan, 2023.