Packing designs with large block size

Andrea Burgess[1], Peter Danziger[2], Daniel Horsley[3], Muhammad Tariq Javed[2]

[1] University of New Brunswick, Saint John, Canada
[2] Toronto Metropolitan University, Toronto, Canada
[3] Monash University, Melbourne, Australia

Given positive integers v, k, t and λ with $v \geq k \geq t$, a *packing design* $\mathrm{PD}_\lambda(v,k,t)$ is a pair $(V,\mathcal{B})$, where V is a v-set and $\mathcal{B}$ is a collection of k-subsets of V such that each t-subset of V appears in at most λ elements of $\mathcal{B}$. The maximum size of a $\mathrm{PD}_\lambda(v,k,t)$ is called the *packing number* and denoted $\mathrm{PDN}_\lambda(v,k,t)$.

We prove that for a positive integer n, $\mathrm{PDN}_\lambda(v,k,t) = n$ whenever $nk-(t-1)\binom{n}{\lambda+1} \leq \lambda v < (n+1)k-(t-1)\binom{n+1}{\lambda+1}$. For fixed t and λ, this determines the value of $\mathrm{PDN}_\lambda(v,k,t)$ when k is large with respect to v. By showing that if no point appears in more than three blocks, the blocks of a $\mathrm{PDN}_2(v,k,2)$ can be directed so that no ordered pair appears more than once, we also extend our results to directed packings with index $\lambda = 1$ and strength $t = 2$.