First-Fit Coloring of Forests in Random Arrival Model

B. Bosek[1,2], G. Gutowski[1], M. Lasoń[3], J. Przybyło[4]

[1] Institute of Theoretical Computer Science, Faculty of Mathematics and Computer Science, Jagiellonian University, Kraków, Poland
[2] Institute of Informatics, University of Warsaw, Poland
[3] Institute of Mathematics of the Polish Academy of Sciences, Warszawa, Poland
[4] AGH University of Krakow, Faculty of Applied Mathematics, Kraków, Poland

We study the performance of the First-Fit coloring algorithm on forests in the random arrival model. While this algorithm is known to use $\Theta(\log n)$ colors in the worst-case (adversarial) online model, its average-case performance under a random vertex permutation has been less understood.

We close this gap by providing tight asymptotic bounds. We show that for any forest with n vertices, the expected number of colors used by First-Fit is at most $(1+o(1))\frac{\ln n}{\ln\ln n}$. Furthermore, we prove this bound is optimal by constructing a family of forests that requires $(1-o(1))\frac{\ln n}{\ln\ln n}$ colors in expectation. Our result precisely characterizes the performance of First-Fit for this graph class, showing a modest but significant gain over the adversarial setting.

References

[1] B.Bosek, G.Gutowski, M.Lasoń, J.Przybylo, First-Fit Coloring of Forests in Random Arrival Model. *MFCS 2024*, 33:1-33:10.