On random regular graphs and the Kim-Vu Sandwich Conjecture

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The random regular graph $G_d(n)$ is selected uniformly at random from all d-regular graphs on n vertices. This model is a lot harder to study than the Erdős-Renyi binomial random graph model $G(n,p)$ as the probabilities of edges being present are not independent. However, in the regime $d \gg \log n$, various graph properties including Hamiltonicity and chromatic number were shown (with hard work) to be the same in $G_d(n)$ as in $G(n,p)$ with $np = d$. This inspired Kim and Vu [2] to conjecture that when $d \gg \log n$ it is possible to 'sandwich' the random regular graph $G_d(n)$ between two Erdős-Renyi random graphs with similar edge density. A proof of the conjecture would immediately imply many results about monotone graph properties of $G_d(n)$ in this dense regime, and would unify all the previous separate hard-won results.

Various authors have proved weaker versions of this conjecture with incrementally improved bounds on d. The previous state of the art was due to Gao, Isaev and McKay [1] who proved the conjecture for $d \gg \log^4 n/(\log\log n)^3$. I will talk about our new improvement of this result.

References

[1] P. Gao, M. Isaev, and B. McKay. Kim-Vu's sandwich conjecture is true for $d \geq \log^4 n$. *arXiv preprint arXiv:2011.09449*, 2020.

[2] J. H. Kim, V. H. Vu, Sandwiching random graphs: universality between random graph models, *Advances in Mathematics* 188(2):444–469, 2004.