On proper secondary and multiple dominating sets

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Let $k \geq 1$ be an integer. A subset $D \subset V(G)$ is $(1,k)$-dominating if for every vertex $v \in V(G) \setminus D$ there exist vertices $u, w \in D$ such that $uv \in E(G)$ and $d_G(v, w) \leq k$. If $k = 1$, then we obtain the definition of $(1,1)$-dominating sets, which are also known as 2-dominating sets. If $k = 2$, then we have the concept of $(1,2)$-dominating sets.

In [1] Michalski et al. introduced the concept of proper $(1,2)$-dominating sets to distinguish $(1,2)$-dominating sets from $(1,1)$-dominating sets. Formally, a *proper $(1,2)$-dominating set* is a $(1,2)$-dominating set that is not $(1,1)$-dominating. Basing on this idea, we considered proper $(1,3)$-dominating sets. Moreover, in [2, 3] proper l-dominating sets i.e. l-dominating sets which are not $(l + 1)$-dominating were defined and studied.

In this talk we present some results concerning proper secondary and multiple dominating sets, in particular we focus on the problem of their existence. Moreover, we show relations between parameters of these types of domination.

References

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